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MULTI–MODEL FORMULATIONS FOR COMPRESSIBLE VISCOUS PLASMA FLOWS IN MPD–ACCELERATORS

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Abstract. *We present a method for the numerical treatment of high–enthalpy compressible viscous plasma flows inside magnetoplasmadynamic self–field accelerators. Such flows are modelled by the compressible Navier–Stokes equations, which are extended by partial differential equations describing the electromagnetic field. Since the numerical solution of the governing system is extremely expensive, the latter is locally reduced to a simplified model, and the problem (which is homogeneous in nature) is modelled here in a heterogeneous fashion: the Navier–Stokes equations (extended under the influence of an arc discharge) in the near field of the MPD–accelerator are coupled with the extended Euler equations of inviscid plasma flow in the complementary far–field region. These models are coupled by the continuity of the normal fluxes as transmission condition at the artificial coupling boundary. Consequently, a boundary layer phenomenon appears in the vicinity of the interface, where the approximate Navier–Stokes/Euler coupled solution exhibits jumps depending on the magnitude of the viscosity terms neglected in the far field. We present numerical results of the heterogeneous domain decomposition.*

1 INTRODUCTION

Flying to the planets and moons of the solar system in a short time requires rocket thrusters with a thrust of several Newton, high exhaust speed and low fuel consumption. Magnetoplasmadynamic rocket propulsion may be able to fulfil these requirements at relatively low costs, and, therefore, MPD self-field thrusters are of great interest for interplanetary missions. However, the high power self-field MPD thrusters still require basic mathematical, experimental and numerical research in order to avoid flow instabilities and to improve the thruster performance and lifetime. These axisymmetric devices use an arc discharge between two electrodes to heat a cold gas up to several ten-thousand Kelvin. The plasma carries an electric current of density $\mathbf{j}$ which induces an azimuthal magnetic field $\mathbf{B}$, see Figure 1 (left). Consequently, the plasma flow is thermally expanded and also accelerated by the (electromagnetic) Lorentz forces $\mathbf{j}\times\mathbf{B}$, and so a very high exhaust velocity is achieved, which is necessary for efficiently propelling spacecrafts. Due to the low arc voltage and the relatively low atomic weight we consider argon as propellant for MPD thrusters. The plasma flow is accelerated into a test tank in the laboratory, see Figure 1 (right) for an axial section of the geometry.

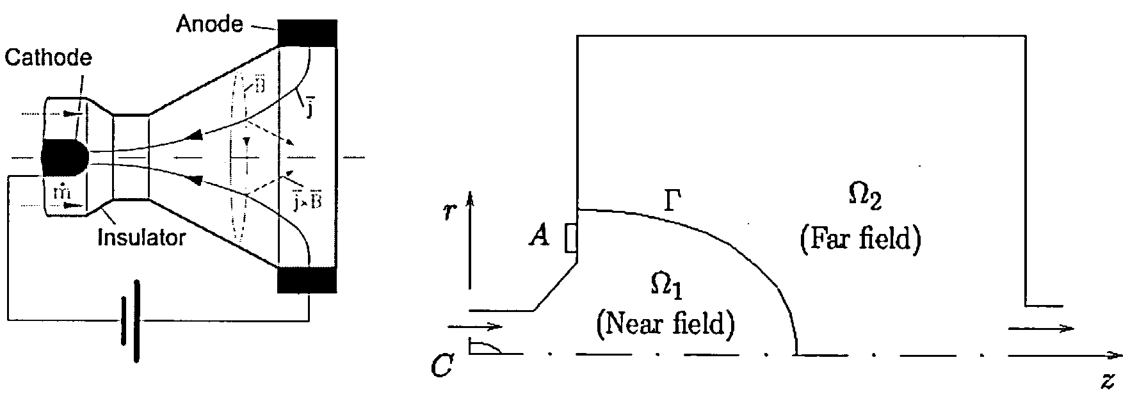

Figure 1: MPD thruster (left). Decomposition of the computational domain (right)

The heavy particle flow (consisting of argon atoms Ar^0 and ions Ar^{1+}, Ar^{2+}) is described by the conservation equations for mass, momentum and energy, resulting in the compressible Navier–Stokes equations, extended by the influence of the arc discharge. For the electron field we discretize the conservation equation for electron and internal energy. In addition, Ohm's law for plasmas and the Maxwell equations of classical electrodynamics are considered, and reaction equilibrium, thermal non-equilibrium (two-fluid model), and laminar flow are assumed. The conservation equations are of hyperbolic–parabolic type and include strong source terms.

Because of the tremendeous computational costs, the direct numerical treatment of this complex mathematical model in the whole configuration MPD–thruster / test tank is not desirable. In order to provide an efficient solution algorithm, we apply the idea of heterogeneous domain decomposition, by replacing the original system with simpler models within subregions of the computational domain. Nevertheless, the derivation of the simplified models needs to incorporate as much as possible the physical properties of the considered plasma flow. We start by decomposing the computational domain into a rather small near field Ω_1 containing the accelerator, and the complementary far field Ω_2 corresponding to the test tank, as shown in Figure 1 (right). The choice of the interface Γ between the model zones (here, a quarter of an ellipse) should also take into consideration the physics of the problem. The underlying idea is to discretize the governing system with the extended Navier–Stokes equations only in the near field Ω_1, and to couple the approximate solution to the solution of the simplified model of the (extended) Euler equations with vanishing magnetic field, which is employed in Ω_2.

A first heterogeneous Navier–Stokes/Euler coupling for plasma flows is presented in [2], where the interface is chosen as the quarter of a circle. This investigation represented a first successful attempt to discretize the compressible plasma flow within the whole configuration, giving first information about the approximate coupled solution. While the Euler model has proved to be a good candidate for the description of the plasma flow in Ω_2, however, the approximate solution exhibits jumps at the coupling boundary, depending on the magnitude of the viscosity terms neglected in the far field. Taking into account that the purely hyperbolic (extended) Euler model has been used too close to the MPD–thruster, we have continued our research in [3], where we present a second heterogeneous domain decomposition, by using three model zones where three different mathematical models of extended systems of conservation laws are employed. In this contribution we continue the analysis of the approximate coupled solution. Results of our numerical computations, performed here by using an elliptical interface Γ, are presented.

Since the solution of the original (Navier–Stokes) problem should satisfy the **natural** transmission conditions (i.e. continuity of the solution and of the total normal flux) across the interface, the approximate Navier–Stokes/Euler solution needs to be **corrected** in the inviscid Euler region by special terms accounting for the loss of continuity and maintaining the continuity of the normal flux. A boundary layer correction for a simplified one–dimensional transmission problem is presented in [4] in the framework of singular perturbation theory. A boundary layer correction method for a two–dimensional Navier–Stokes/Euler coupled problem is in preparation [5].

2 CONSERVATION EQUATIONS

The axisymmetric plasma flow is described in cylindrical coordinates by the function

$$\mathbf{W} = \mathbf{W}(r, z;\, t) := [\,\mathbf{w}, p_{H}, T_{H};\ \mathbf{w}_e;\ \mathbf{w}_{EB}\,]^{\top}(r, z;\, t), \quad (r, z) \in \Omega,\ t \in [0, T].$$

Here, the component $\mathbf{w} = (\rho, \rho v_r, \rho v_z, E_H)^{\top}$ of $\mathbf{W}$ collects the conservative variables with the density ρ, the velocity vector $\mathbf{v} = (v_r, v_z)^{\top}$, and the energy of the heavy particles E_H. The pressure and the temperature of the heavy particles are denoted by p_H and T_H, respectively. The function $\mathbf{w}_e = (e_{ei}, p_e, T_e)^{\top}$ describes the electron component of the plasma, with e_{ei} containing the electron and the ionization energy, and with p_e and T_e representing the pressure and the temperature of the electron component, respectively. Finally, $\mathbf{w}_{EB} = (\mathbf{E},\mathbf{B},\mathbf{j})^{\mathsf{T}}$ contains the electromagnetic field $(\mathbf{E},\mathbf{B})$ and the electric current density $\mathbf{j}$.

2.1 Mathematical modelling of the near field

The **heavy–particle flow** is modelled by the compressible Navier–Stokes equations, extended by the influence of an arc discharge. Written in conservative form, these equations take in cylindrical coordinates the form

$$\frac{\partial \mathbf{w}_1}{\partial t} + \mathrm{div}_{(r,z)}\mathbf{F}(\mathbf{W}_1) = \mathrm{div}_{(r,z)}\mathbf{R}(\mathbf{w}_1, \nabla_{(r,z)}\mathbf{w}_1) + \mathbf{G}(\mathbf{W}_1) \quad \text{in} \quad \Omega_1 \times [0,T], \tag{1}$$

with $\mathrm{div}_{(r,z)}$ denoting the divergence operator in cylindrical coordinates, given by $\mathrm{div}_{(r,z)}\mathbf{a} = \frac{1}{r}\frac{\partial}{\partial r}(r a_r) + \frac{\partial a_z}{\partial z}$. The function $\mathbf{F}$ contains the purely convective part from fluid dynamics and an electromagnetic pressure term derived from the source terms. We represent $\mathbf{F}$ as

$$\mathbf{F} = (\mathbf{F}_r, \mathbf{F}_z)\,(\mathbf{W}) = (\mathbf{f}_r, \mathbf{f}_z)(\mathbf{w}, \mathbf{w}_e) + (\mathbf{g}_r, \mathbf{g}_z)(\mathbf{w}_{EB}), \tag{2}$$

where, with the purely azimuthal magnetic field $\mathbf{B} = (0, B_\varphi, 0)^{\mathsf{T}}$ and with the magnetic permeability of vacuum $\mu_0 > 0$,

$$\begin{aligned}
\mathbf{f}_r(\mathbf{w}, \mathbf{w}_e) &:= \left(\rho v_r,\ \rho v_r^2 + (p_H + p_e),\ \rho v_r v_z,\ [E_H + (p_H + p_e)]\, v_r\right)^{\mathsf{T}}, \\
\mathbf{f}_z(\mathbf{w}, \mathbf{w}_e) &:= \left(\rho v_z,\ \rho v_z v_r,\ \rho v_z^2 + (p_H + p_e),\ [E_H + (p_H + p_e)]\, v_z\right)^{\mathsf{T}}, \\
\mathbf{g}_r(\mathbf{w}_{EB}) &:= (0,\ B_\varphi^2,\ 0,\ B_\varphi^2 v_r)^{\mathsf{T}}/(2\mu_0),\ \ \mathbf{g}_z(\mathbf{w}_{EB}) := (0,\ 0,\ B_\varphi^2,\ B_\varphi^2 v_z)^{\mathsf{T}}/(2\mu_0).
\end{aligned}$$

The viscous terms are collected in the function $\mathbf{R} = (\mathbf{R}_r, \mathbf{R}_z)(\mathbf{w}, \nabla_{(r,z)}\mathbf{w})$ where

$$\begin{aligned}
\mathbf{R}_r(\mathbf{w},\nabla_{(r,z)}\mathbf{w}) &:= \left(0,\ \tau_{rr},\ \tau_{rz},\ \tau_{rr}v_r + \tau_{rz}v_z + \lambda_H\, \partial T_H/\partial r\right)^{\mathsf{T}}, \\
\mathbf{R}_z(\mathbf{w},\nabla_{(r,z)}\mathbf{w}) &:= \left(0,\ \tau_{zr},\ \tau_{zz},\ \tau_{zr}v_r + \tau_{zz}v_z + \lambda_H\, \partial T_H/\partial z\right)^{\mathsf{T}},
\end{aligned}$$

with the heat conductivity $\lambda_H > 0$ of the heavy–particle flow, and with

$$\tau_{rr} = \mu\left[2\frac{\partial v_r}{\partial r} - \frac{2}{3}\,\mathrm{div}\,\mathbf{v}\right],\ \tau_{rz} = \tau_{zr} = \mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right],\ \tau_{zz} = \mu\left[2\frac{\partial v_z}{\partial z} - \frac{2}{3}\,\mathrm{div}\,\mathbf{v}\right]$$

defining the components of the viscous part of the stress tensor; $\mu > 0$ represents the viscosity coefficient.

The function $\mathbf{G}$ contains the electromagnetic force and heat terms as well as a quantity describing the heat transfer due to the collisions between the plasma components:

$$\mathbf{G}(\mathbf{W}) := \left(0,\ \frac{1}{r}\left[p_H + p_e - \tau_{\varphi\varphi} - \frac{B_\varphi^2}{2\mu_0}\right],\ 0,\ \left(p_e + \frac{B_\varphi^2}{2\mu_0}\right)\mathrm{div}\,\mathbf{v} - \frac{v_r}{r}\frac{B_\varphi^2}{\mu_0} + \mathcal{H}_{coll}\right)^{\mathsf{T}} \tag{3}$$

where $\tau_{\varphi\varphi} := \frac{2}{3}\mu\big(2\frac{v_r}{r} - \frac{\partial v_r}{\partial r} - \frac{\partial v_z}{\partial z}\big)$. The effect of the collisions is modelled by the quantity

$$\mathcal{H}_{coll} := \sum_{\nu=0}^{2} n_\nu n_e \alpha_{e\nu}(T_e - T_H), \tag{4}$$

where n_ν ($\nu = 0, 1, 2$) and n_e are the densities of the heavy particles and of the electrons, respectively, and $\alpha_{e\nu}$ are heat transfer coefficients.

Since the electrons are characterized by very small mass and momentum, but significant energy, the solely conservation law considered for their approximation is the **equation for the electron and internal energy**, which is in $\Omega_1 \times [0, T]$ given by

$$\frac{\partial e_{ei}}{\partial t} + \operatorname{div}(e_{ei}\mathbf{v}) - \operatorname{div}(\lambda_{ei}\nabla T_e) = -p_e \operatorname{div}\mathbf{v} + \frac{5}{2}\frac{k}{e}\mathbf{j}\cdot\nabla T_e - \frac{1}{n_e e}\mathbf{j}\cdot\nabla p_e - \mathcal{H}_{coll} + \frac{|\mathbf{j}|^2}{\sigma}. \tag{5}$$

Here, λ_{ei} denotes the heat conductivity for the electron component of the flow, k is the Boltzmann constant, and σ is the electric conductivity.

Finally, for the description of the **electromagnetic field**, we use the Maxwell equations and Ohm's law for plasmas, which read as

$$\operatorname{rot}\mathbf{B} = \mu_0\mathbf{j}, \ \operatorname{rot}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \ \operatorname{div}\mathbf{B} = 0; \quad \mathbf{E} = \frac{\mathbf{j}}{\sigma} - \mathbf{v}\times\mathbf{B} + \beta\,\mathbf{j}\times\mathbf{B} - \beta\,\nabla p_e$$

with β denoting the Hall parameter. The functions $\mathbf{E}$ and $\mathbf{j}$ can be eliminated, and we obtain the discharge equation

$$\frac{\partial B_\varphi}{\partial t} + \operatorname{div}(B_\varphi\mathbf{v}) = \frac{B_\varphi v_r}{r} - \operatorname{rot}\Big(\frac{\operatorname{rot}\mathbf{B}}{\mu_0\sigma} + \beta\frac{\operatorname{rot}\mathbf{B}}{\mu_0}\times\mathbf{B} - \beta\nabla p_e\Big)_{,\varphi} \quad \text{in} \quad \Omega_1\times[0,T]. \tag{6}$$

Here, $\mathbf{a}_{,\varphi}$ denotes the azimuthal component a_φ of the vector $\mathbf{a} := (a_r, a_\varphi, a_z)$. In addition, we need the constitutive thermodynamic relations between the unknowns, namely the Clapeyron relation and the definition of the energy. These yield the following equations of state for the heavy particle flow:

$$p_H = \sum_{i=0,1,2} n_i k T_H, \quad p_H = (\kappa - 1)\Big[E_H - \frac{1}{2}\rho|\mathbf{v}|^2\Big], \tag{7}$$

and for the electron field

$$p_e = n_e k T_e, \ e_e = \frac{3}{2} n_e k T_e. \tag{8}$$

In addition to the system (1)–(8) we require the initial conditions

$$\mathbf{W}_1(r, z; 0) := \mathbf{W}_1^0(r, z) \quad \text{for} \quad (r, z) \in \Omega_1 \tag{9}$$

and the boundary conditions for viscous flows,

$$\mathcal{B}_{visc}(\mathbf{W}_1) = 0 \quad \text{on} \quad \partial\Omega_1 \setminus \Gamma. \tag{10}$$

For the heavy–particle flow, the boundary operator $\mathcal{B}_{visc}$ is defined by

$$\mathbf{v} = 0 \quad \text{and} \quad T_H = T_{H,wall} \quad \text{on} \quad \partial\Omega_{1,\, solid\ wall}\,, \tag{11}$$

as well as by the condition of axisymmetric flow

$$\frac{\partial \mathbf{v}}{\partial r} = \frac{\partial T_H}{\partial r} = 0 \quad \text{on} \quad \Gamma_S. \tag{12}$$

Furthermore, we assume that the electrons behave adiabatically on the boundary, i.e.,

$$\frac{\partial T_e}{\partial \mathbf{n}} = 0 \quad \text{on} \quad \partial\Omega_1 \setminus \Gamma. \tag{13}$$

Similar to (13), we require the condition of axisymmetric electron field

$$\frac{\partial T_e}{\partial r} = 0 \quad \text{on} \quad \Gamma_S. \tag{14}$$

For the electromagnetic component, we first impose the classical boundary condition $\mathbf{E}-(\mathbf{E}\cdot\mathbf{n})\mathbf{n} = \mathbf{0}$ at the electrodes. With the help of the Maxwell equations, this condition can be simplified so that it permits the calculation of the time updating of the magnetic field component B_φ at the correponding boundary cells. On the other parts of the solid walls (insulator), Ampere's law leads to $B(r,z)=\frac{\mu_0}{2\pi r}I\ (r>0)$, where I is the total thruster current. From the assumption of axisymmetry we derive the condition $\mathbf{B}=\mathbf{0}$ on Γ_S, which is employed in the calculations.

In order to close the system, one also needs boundary conditions at the artificial far–field interface Γ. The needed boundary data are obtained by using the approximate solution of the simplified model of extended Euler equations in the far field $\Omega_2 := \Omega \setminus \Omega_1$.

2.2 Simplified modelling of the far field

In the subdomain Ω_2, whose area is essentially larger than that of the near field Ω_1, covering the main part of the tank, we assume the plasma flow to be inviscid, i.e., the shear stresses $\tau_{rr}, .., \tau_{zz}$ and the heat conduction terms $\lambda_H \partial_{r(z)} T_H$, defining the quantity $\mathbf{R}$, are strongly dominated by the convective part from (2). We also assume that the magnetic field $\mathbf{B}$ vanishes identically in Ω_2. The governing system takes the reduced form

$$\frac{\partial \mathbf{w}_2}{\partial t} + \mathrm{div}_{(r,z)}(\mathbf{f}_r, \mathbf{f}_z)(\mathbf{w}_2, \mathbf{w}_{e,2}) = \mathbf{H}(\mathbf{w}_2, \mathbf{w}_{e,2}) \quad \text{in} \quad \Omega_2 \times [0,T], \tag{15}$$

with the simplified source term

$$\mathbf{H}(\mathbf{w}_2, \mathbf{w}_{e,2}) := \left[0,\ \frac{p_H + p_e}{r},\ 0,\ p_e \,\mathrm{div}\,\mathbf{v} + \mathcal{H}_{coll}\right]. \tag{16}$$

Furthermore, as a consequence of $\mathbf{j} = \mathrm{rot}\mathbf{B}/\mu_0 \equiv 0$ in Ω_2, the equation of conservation for the electron and the ionization energy (5) is in $\Omega_2 \times [0,T]$ reduced to

$$\frac{\partial e_{ei}}{\partial t} + \mathrm{div}\,(e_{ei}\mathbf{v}) - \mathrm{div}\,(\lambda_{ei}\nabla T_e) = -p_e \,\mathrm{div}\,\mathbf{v} - \mathcal{H}_{coll}. \tag{17}$$

In addition to the state equations (7), (8), and to the initial conditions of the form (9), we impose boundary conditions

$$\mathcal{B}_{invisc}(\mathrm{W}_2) = 0 \quad \text{on} \quad \partial\Omega_2 \setminus \Gamma, \tag{18}$$

which are typical for inviscid Euler flow. The boundary conditions defined by the operator $\mathcal{B}_{invisc}$ include, in particular, the homogeneous Neumann boundary condition of non-penetration, as well as adiabatic boundary conditions in the form

$$\mathbf{v} \cdot \mathbf{n} = 0, \; \frac{\partial T_H}{\partial \mathbf{n}} = 0, \; \frac{\partial T_e}{\partial \mathbf{n}} = 0 \text{ on } \partial\Omega_{2,\, solid\ wall}. \tag{19}$$

On Γ_S, the symmetry conditions (12) and (14) are employed.

3 TRANSMISSION CONDITIONS

From the mathematical point of view, a crucial issue in the heterogeneous domain decomposition is the construction of matching conditions across the interface separating the model zones. The derivation of appropriate transmission conditions between the viscous and the inviscid solutions has to be done in such a way, that on one hand, the fundamental physical laws are respected, and on the other hand, the resulting coupled problem is well-posed and consistent with the full original problem. The continuity of the characteristic variables could be chosen as transmission condition, but according to the theory of hyperbolic equations, this can be required only across that part of the interface, where the corresponding characteristics enter the hyperbolic region, see e.g. [9, 10, 1, 6]. In accordance with the conservation laws, the continuity of the normal flux yields a transmission condition on the complete interface Γ: the total flux associated with the full model in Ω_1 (containing the inviscid as well as the viscous contributions) is set equal to the normal inviscid flux, that results from the simplified equations in Ω_2:

$$\begin{aligned} -[\mathbf{R}_r(\mathbf{w}_1, \nabla\mathbf{w}_1)\, n_r + \mathbf{R}_z(\mathbf{w}_1, \nabla\mathbf{w}_1)\, n_z] + [\mathbf{f}_r(\mathbf{w}_1, \mathbf{w}_{e,1}) + \mathbf{g}_r(\mathbf{w}_{EB,1})]\, n_r \\ +[\mathbf{f}_z(\mathbf{w}_1, \mathbf{w}_{e,1}) + \mathbf{g}_z(\mathbf{w}_{EB,1})]\, n_z = \mathbf{f}_r(\mathbf{w}_2, \mathbf{w}_{e,2})\, n_r + \mathbf{f}_z(\mathbf{w}_2, \mathbf{w}_{e,2})\, n_z. \end{aligned} \tag{20}$$

Note that, imposing this transmission condition, we neglect the viscous fluxes and the heat transfer corresponding to the heavy-particle flow. As a consequence, the solutions of the coupled problem may exhibit jumps at the interface, depending on the magnitude of the viscosity and heat transfer terms neglected in Ω_2. Since the solution of the original problem should satisfy the **natural** transmission conditions at the artificial interface (i.e. continuity of the solution and of the total normal flux), the approximate extended Navier–Stokes / extended Euler solution can only be a first approximation and needs to be corrected by special terms accounting for the loss of continuity and maintaining the continuity of the normal flux.

In order to assure the conservation of the electron heat flux across the interface, we impose the continuity of the co-normal derivative of the electron temperature:

$$\left[\lambda_{ei,1}\frac{\partial T_{e,1}}{\partial \mathbf{n}}\right](r,z) = \left[\lambda_{ei,2}\frac{\partial T_{e,2}}{\partial \mathbf{n}}\right](r,z) \quad \text{for all} \quad (r,z) \in \Gamma. \tag{21}$$

Finally, we require $B_\varphi \equiv 0$ on Γ.

4 NUMERICAL ASPECTS AND RESULTS

The steady-state solution of the coupling boundary-transmission value problem with the MPD system is obtained by time stabilization from time-stepping unsteady (extended) approximations on unstructured meshes with dual cells. The extended conservation laws (1) and (15), the electron and the ionization energy equations (5) and (17) as well as the discharge equation (6) are solved on an unstructured, dual mesh by using a second-order finite volume upwind scheme based on explicit Euler time-stepping, [7, 8]. The test case chosen was the hot anode thruster HAT with a current of 2000 A and an argon mass flow rate of 0.8 g/s.

The full computational domain including the near field of the MPD accelerator and the far field corresponding to the tank, is shown in Figure 2. The area of the far field is about 24 times larger than that of the near field, emphasizing the necessity of simplifying the mathematical model in Ω_2.

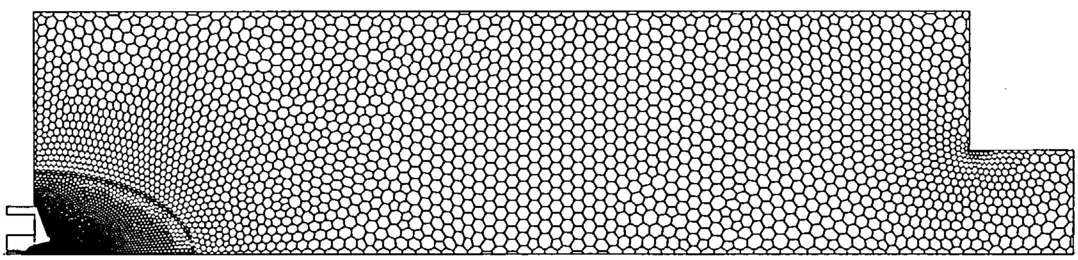

Figure 2: Full computational domain.

Figure 3 presents the isolines of the components v_z and v_r of the velocity field, considered in the longitudinal and radial directions, respectively. The isolines of v_z give an overall impression of the plasma flow: The plasma is accelerated in the MPD accelerator, it is then expanded into the tank, and a part of it flows out of the tank at the far right.

However, it turns out (see also [3]) that this simplified model provides a satisfactory numerical solution almost in the whole far field Ω_2, except in the immediate vicinity of Ω_1. A detailed section of the approximate coupled solution at the artificial boundary Γ is presented in Figures 4 and 5. The isolines of the longitudinal velocity show that the coupling method works well for the central plasma jet, v_z passing smoothly the interface

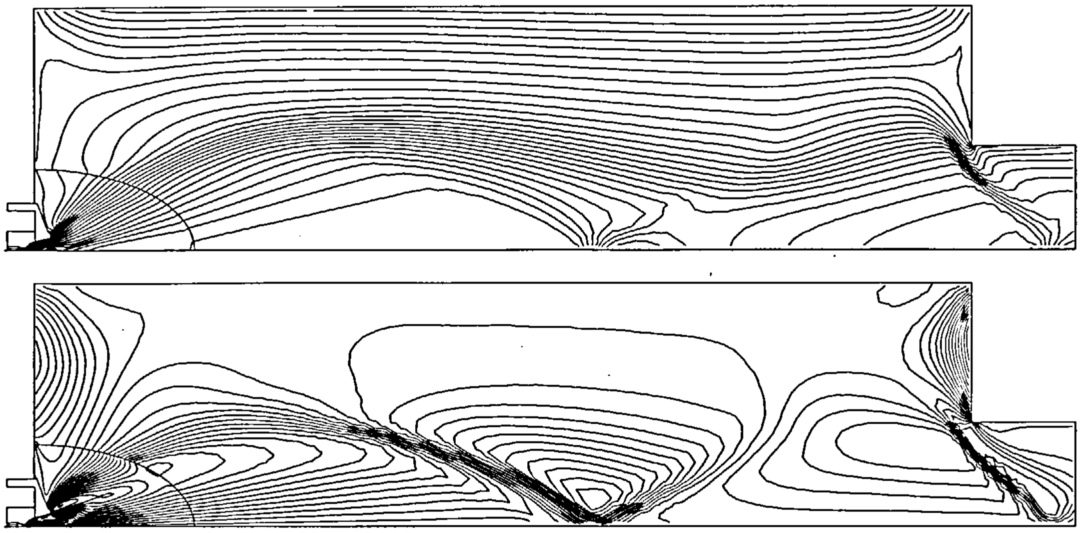

Figure 3: Isolines of the longitudinal velocity v_z (left) and of the radial velocity v_r (right)

Γ there. However, farther away from the centerline, the isolines show small jumps and have kinks across the interface. A similar behavior is shown also by v_r: the corresponding isolines pass the coupling boundary smoothly up to one nozzle radius above the centerline, whereas in the recirculation zone at the left (insulator) boundary on top of the anode, the approximate solution admits jump discontinuities and the smoothness property is lost.

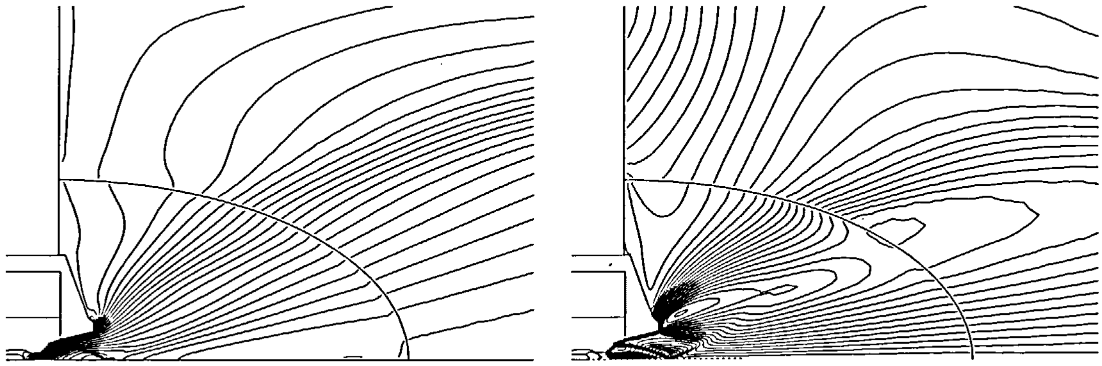

Figure 4: Isolines of the longitudinal velocity v_z (left) and of the radial velocity v_r (right)

Figure 5 (left) shows the effect of neglecting the heat conduction terms $\lambda_{II}\partial_{r(z)}T_{II}$ corresponding to the heavy–particle flow. The isolines of the heavy–particle temperature T_{II} behave smoothly across the part of Γ contained in the central plasma jet, but T_{II} admits jumps across the rest of Γ. Moreover, the hyperbolic character of the model employed leads to a nonphysical shock wave there. Let us outline that T_H also satisfies different boundary conditions at the left (insulator) boundary on top of the anode, namely:

$$\begin{aligned} T_H &= T_{H,wall} \text{ on } \partial\Omega_{1,\ solid\ wall} \quad \text{(Dirichlet)}, \\ \frac{\partial T_H}{\partial n} &= 0 \text{ on } \partial\Omega_{2,\ solid\ wall} \quad \text{(homogeneous Neumann)}. \end{aligned} \tag{22}$$

Finally, Figure 5 (right) shows the isolines of the electron temperature T_e for the coupled solution.

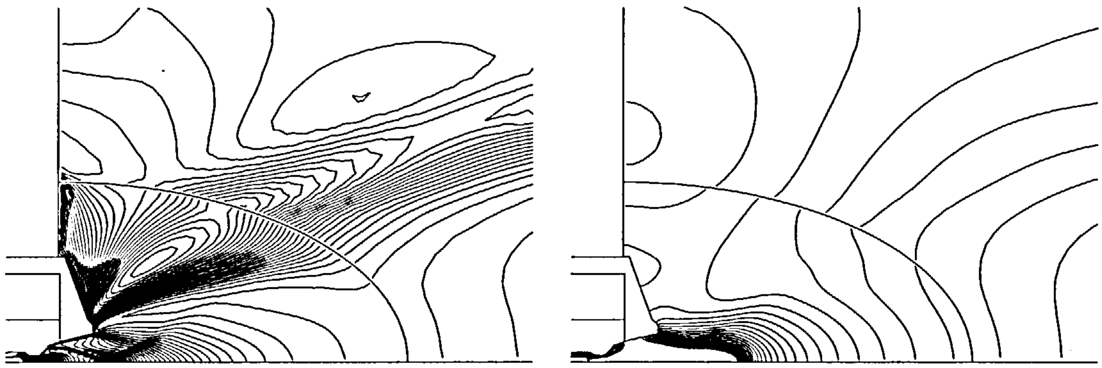

Figure 5: Isolines of the heavy-particles temperature T_H (left) and of the electron temperature T_e (right)

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