A WITTGENSTEINIAN SOLUTION TO THE SORITES

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It is difficult to begin at the beginning. And not try to go further back.
(Wittgenstein, *On Certainty*, § 471)

Gottlob Frege thought that concepts could not be vague. In his *Basic Laws of Arithmetic* (ii, § 56) he wrote:

> To a concept without sharp boundaries there would correspond an area that had not a sharp boundary-line all round, but in places just vaguely faded away into the background. This would not really be an area at all; and likewise a concept that is not sharply defined is wrongly termed a concept.

Similar negative attitudes to vagueness, according to which it is an indication of defective logic or knowledge, were held by Russell (1923), the early Wittgenstein (1922, 3.251), and most other logicians of their age.

In his *Philosophical Investigation* Wittgenstein set out to defend the acceptability of vague concepts. In sections 65-71, 76-77, 79-80, 88 and in other places he discusses, among other things, various sorts of vague concepts, and shows how we can explain their meaning, how useful they may be, and other features they have. Criticizing Frege, he writes (§ 71):

> Frege compares a concept to a region and says that a region without clear boundaries cannot be called a region at all. This presumably means that we cannot do anything with it. — But is it senseless to say: "Stay roughly here"? Imagine that I were standing with someone in a city square and said that. As I say it, I do not bother drawing any boundary, but just make a pointing gesture – as if I were indicating a particular spot. And this is just how one might explain what a game is. One gives examples and

intends them to be taken in a particular way.[1]

The concept of a game, like many other concepts, is vague, but this does not make it in any way defective.

Following Wittgenstein's work, most philosophers have gradually come to acknowledge vague concepts as legitimate. It is now common to think that, from a metaphysical point of view, there is no fault in people being, say, neither definitely young nor definitely not young. Epistemically, vague concepts are not an indication of a confused or an otherwise defective knowledge—as is attested by the prevalence in science of vague concepts such as 'solid' or the concepts of the various species. Vague concepts are ubiquitous, and there is nothing problematic in that.

Or *almost* nothing: there is still one remaining discontent, a logical one, since some vague concepts give rise to the Sorites, the paradox of the heap.

Wittgenstein himself never mentions the Sorites in his extant writings. Still, an attempt to resolve the paradox might be claimed to be Wittgensteinian in spirit. As the title of this paper discloses, I attempt in it to resolve the paradox, and in a manner which I consider Wittgensteinian. I hope this characterization would be justified as we proceed; but whether or not it would, the success of my attempt should of course be judged irrespective of whether it is indeed Wittgensteinian. So let us start with some classifications of concepts.

A concept is vague just in case there are items of which it is indeterminate whether it applies to them or not. For instance, 'religion' is vague, since Buddhism is neither definitely a religion nor definitely not one (Buddhism shares many characteristics with Western religions, yet it has no god). So is 'heap', since some collections of grains are not definitely heaps, yet they are not definitely non-heaps either.

Now if we remove one grain from a heap of grains, the remaining collection of grains is still a heap. It thus seems to follow that if we repeat this process again and again, the remaining collection of grains will always be a heap. However, we shall end up with a single grain, which is no heap. We have a paradox, the paradox of the heap. Can this paradox be resolved?

The concept of religion, it seems, despite being vague, does not give rise to the Sorites. The vagueness involved in 'religion', as W. P. Alston put it (1967, pp. 219-20), is *combinatory*:

[1] I am following the draft of Hacker and Schulte's forthcoming revised translation of the *Philosophical Investigations*, which they very kindly made available to me.

it is indeterminate how many characteristics out of a given group a practice should have in order to count as a religion. By contrast, the vagueness involved in 'heap' is *degree* vagueness: the boundary along some dimension, or dimensions, between collections that are heaps and collections that are not, is indeterminate. And there probably are other varieties of vagueness as well: for instance, the concept of *number*, as Wittgenstein claimed (PI § 67), is also vague, but its vagueness is not degree vagueness, and probably not combinatory vagueness either. Returning to degree-vague concepts, they are said to be *tolerant*: for these concepts there is 'a notion of a degree of change too small to make any difference' to their applicability (Wright 1976, 156). Concepts whose vagueness is degree vagueness give rise to the Sorites; vague concepts of other sorts give rise to the paradox only if some of the characteristics they involve are degree-vague.

(It would be noticed that while much of contemporary literature identifies vagueness with degree vagueness, I consider a wider variety of concepts vague—again following Wittgenstein, as well as Alston and other earlier sources. This current one-sided diet of examples (cf. *PI* 593) impoverishes, I believe, our understanding of the semantic phenomenon of vagueness. Nowadays one often finds vagueness characterized by means of tolerance or related ideas (e.g., Greenough's epistemic tolerance (Greenough 2003)), while other kinds of vagueness, such as combinatory vagueness, are not even mentioned. This is probably the result of the preoccupation of the literature with the Sorites.)

Contrary to what is sometimes thought, the Sorites is not due to our inability to distinguish between consequent stages in a series. A salary of 1$ per month is not a high one, while a salary of 1,000,000$ per month is. Moreover, a salary increase of 1$ per month cannot turn a salary into a high one—the Sorites ensues. But all the same, the difference between *n* and *n*+1 dollar salary on the pay-slip is easily discernible. If successive members in a series are indistinguishable as to how *f* they are, while members several steps removed from each other *are* distinguishable in that respect, then '*f*' cannot in practice be replaced by a sharp (i.e., non-vague) concept that would coincide with it on its clear applications. But the Sorites arises even for vague concepts that can be practically sharpened, and such vague concepts are as essential to our language and thought as are vague concepts that cannot.

Several suggested solutions to the Sorites exist in the literature, yet none has won wide support. In fact, each suggestion has received powerful criticisms, which most philosophers think have not been satisfactorily met. (Presentations of these suggestions and of their criticisms abound in the literature, so we need not supply them here.[2]) Accordingly, the situation clearly calls for an attempt at a new solution, an attempt to which we shall now proceed. (As I focus on a solution for the Sorites, I shall not discuss other vagueness issues, e.g. second-order vagueness.)

I start by presenting the Sorites argument in as tight a form as I can. I first assume that *a collection of 1,000 grains is a heap*. Clearly, the specific number of grains assumed sufficient for a collection of grains to be a heap is immaterial for the paradox.

Secondly, I assume that *if a single grain is removed from a heap, the remaining collection of grains is still a heap*.

According to the epistemic solution of the Sorites, at some stage, following the removal of a single grain from a heap, the remaining grain-collection is no longer a heap; we just may not or cannot *know* which grain that is. So my second assumption in fact presupposes that the epistemic solution of the paradox is mistaken. Yet I do not find this a fault: I think this assumption is part and parcel of our *understanding* of what a heap is. Similarly, if one *understands* what a child is, if one mastered the criteria for the application of the concept of a child, one knows that in the normal course of events, a person does not stop being a child in a day. This is why the epistemic solution has met with such incredulity: the epistemicist seems to be denying an analytic truth.[3]

I shall now write the propositions that appear in the Sorites argument:

1a	A collection of 1,000 grains is a heap.
1b	If a collection of 1,000 grains is a heap, so is a collection of 999 grains.
2a	A collection of 999 grains is a heap.
2b	If a collection of 999 grains is a heap, so is a collection of

[2] Detailed introductions and surveys, which also include many references, are Read 1994, chap. 7; Sainsbury 1995, chap. 2; Keefe & Smith's introduction to their 1996 reader; and Sainsbury & Williamson 1997. For updated and updating surveys, see Hyde 2005 and Sorensen 2006. Comprehensive examinations of the philosophical debate on vagueness are found in Williamson 1994, Keefe 2000 and Shapiro 2006. Keefe & Smith 1996 contains many of the most influential papers on vagueness.

[3] Cf. Grice and Strawson (1956) on analyticity and understanding. See also (Enoch 2007) for a recent argument against epistemicism, in addition to those found in the literature mentioned in the previous footnote.

998 grains.

…

999b If a collection of 2 grains is a heap, so is a single grain.

1,000a A single grain is a heap.

All the b-propositions follow immediately (by Universal Instantiation) from my second assumption, and therefore are true.

Now since proposition (1a) is true (first assumption), and so is proposition (1b) (from the second assumption), it follows (modus ponens) that proposition (2a) is true. Reiterating this short argument 999 times, it seems to follow that proposition (1,000a), that a single grain is a heap, is true. However—and this is my third and final assumption—*a single grain is not a heap*. We seem to have ended with a contradiction.

Any other form of the Sorites argument that seems to entail the false conclusion either contains a long chain of arguments like the argument above, or derives its seeming validity from such an argument. For instance, the argument:

> 1000 grains make a heap; removing one grain from a heap leaves it a heap. Hence, one grain makes a heap.

seems valid because we think, relying on its second premise, that if we repeat the process of removing one grain 999 times, we should end with a heap (cf. Keefe & Smith [1996], p. 11). To resolve the Sorites it is therefore sufficient to show why the arguments containing a long chain of arguments are invalid.

I now proceed to derive my solution. First, define a *concatenation of arguments* as a sequence of arguments in which the conclusion of each but the last argument is among the premises of the following argument. Such a concatenation yields a *concatenated argument*. Its premises are all the premises of the sequence members that are not conclusions of any previous member; its conclusion is the conclusion of the last argument in the sequence. The Sorites argument formulated above is clearly such a concatenated argument. (In fact, in antiquity a concatenated argument was *called* a Sorites, this paradoxical argument being the best-known example of argument chaining.)

Now, since we have to accept my three assumptions, and since every short argument is valid, it follows that *in some cases a concatenation of many valid arguments yields an invalid argument*. In our case, each argument whose premises are propositions (Na) and (Nb) and whose conclusion is proposition (N+1a) is valid; while the concatenated argument, whose premises are propositions (1a) and (Nb), $1 \leq N \leq 999$, and whose conclusion is proposition (1,000a), is invalid. We can supply a criterion that determines when arguments of this kind are invalid: *A concatenation of many such modus ponens arguments is invalid if it passes the vague boundary between cases where the relevant concept applies and cases where that concept does not apply*. 'To pass the vague boundary' means that some of the interim conclusions are categorical statements about indeterminate cases (i.e., '*a* is *F*', where *a* is a borderline case of '*F*'). Sorites arguments are concatenations of valid arguments that pass vague boundaries, and are therefore invalid. That was my solution.

Readers are bound to protest at this stage: isn't this professed solution merely a restatement of the paradox? For I have not said *where* the Sorites argument goes wrong!—Yet I have not said that, precisely because the *very idea* of my solution is that the argument does not go wrong in any specific single step. Each and every step is valid, but their concatenation is not. It is wrong to look for the specific place where the mistake is made. *We* make a mistake in thinking that if there is no fault in any single step, there must be none in the one-thousand-steps argument as well.

Still, some logicians might claim that I have not explained *why* Sorites arguments are invalid. Presumably, these logicians would demand that the invalidity of Sorites arguments be derived from some other semantic or logical fact (as is done, for instance, by degree-of-truth theory). But such a demand would be misguided. The invalidity of Sorites arguments follows from the fact that these arguments pass vague boundaries of concepts. There is no other more basic yet non-apparent semantic or logical feature of vague concepts or of propositions, from which the invalidity of Sorites arguments is to be derived. In the words of this paper's epigraph, this invalidity is the beginning; one should not try to go further back. Only the unjustified expectation, that any argument would behave as mathematical arguments do (more on this below), made us think that there is something paradoxical which needs explanation in Sorites arguments. Once this expectation is recognized as unjustified, and consequently disappears, all that remains is to supply a criterion that specifies when a concatenation of valid arguments that employ vague concepts is an invalid argument, and little that requires explanation is left. Resolving the Sorites is partly a Wittgensteinian philosophical therapy.

Each step in the Sorites argument is like a grain of sand: logic mirrors reality. Removing one grain of sand from a collection of grains that is a heap leaves it a heap; adding one step to a valid argument leaves it valid. Yet remove many grains, and the remaining collection is no longer a heap; add many steps, and the argument is no longer valid. There is no step which is the *cut* step, neither between heap and non-heap nor between validity and invalidity. Just as what is true of a single grain is not necessarily true of many, what is true of a single step in an argument can be false of many.

Nor would it save the Sorites if it were shifted to the semantic level. One might try to prove the validity of Sorites arguments by constructing a meta-argument, relying on the idea that adding a valid step to a valid argument preserves validity. In that case the premises would be:

1. The argument that has (1a) and (1b) as its premises and (2a) as its conclusion is valid.
2. If an argument that has as its premises (1a), and (1b) to (*N*b), and as its conclusion (*N*+1a) is valid, then adding (*N*+1b) to the premises and substituting (*N*+2a) for the conclusion preserves validity.

It might have seemed that in order to prove the validity of the original Sorites argument, one need only substitute the numbers 1 to 998 for *N*. However, it should now be clear that this meta-argument suffers from the same fault that invalidated the original Sorites argument: many valid meta-steps generate an invalid meta-argument. The semantics and logic of semantic and logical concepts are determined by those of the concepts they describe, and not *vice versa*. Achilles should be wary of tortoise tricks.

Any other variation on the Sorites argument in the attempt to determine a first invalid step or force a false proposition as the conclusion of an apparently sound argument would fail for the same reason, since it would always involve, explicitly or otherwise, a concatenation of valid arguments that crosses a vague boundary.

Contrary to what one might suspect, my solution is not committed to a conception of validity different from the classical one. That an argument is valid means that if its premises are true then, necessarily, its conclusion is also true. The Sorites argument contains, according to my solution, one long invalid argument and many short valid arguments. Let us then examine them in turn.

First, the premises of the long invalid argument are true and its conclusion false, so it does not commit us to a non-classical conception of validity.

Secondly, for a valid short argument to commit us to a different conception of validity, it has to have true premises and a false conclusion. But its premises are of the form 'A collection of n grains is a heap' and 'If a collection of n grains is a heap, so is a collection of n+1 grains' and its conclusion is of the form 'A collection of n+1 grains is a heap'. The second premise is an immediate consequence of our second, conditional assumption, and is therefore true. It follows that there has to be an N such that a collection of N grains is a heap while a collection of $N+1$ grains is not. But since 'heap' is a vague concept, there is no such N. So there is no valid short argument with true premises and false conclusion.

Lastly, let us examine the case of 'middle-sized' Sorites arguments, namely concatenations of valid one-step modus ponens arguments with the final conclusion being about a collection of grains of which it is indeterminate whether it is a heap. In that case, the premises are true and it is indeterminate whether the conclusion is. So it is indeterminate whether the argument is valid, again in accordance with the classical conception of validity.

Hence my solution to the paradox is not implicitly committed to any non-classical conception of validity.

However, this solution does entail some change in our view of the relations between arguments. We used to think that a concatenation of valid arguments is valid, but it turns out that this is not always the case. Such a concatenation always yields a valid argument if all the concepts used in the concatenated arguments are not vague, but it might yield an invalid argument if some of the concepts used are vague (or, more precisely, *degree*-vague). We thought that this is impossible because our picture of the properties of valid arguments was largely derived—as so much else in philosophy—from mathematics; and mathematical arguments do not employ degree-vague concepts. But precisely because of this last fact one should have expected that some characteristics of mathematical arguments might not be shared by arguments that do employ degree-vague concepts when characteristics specific to these concepts are at play. And indeed, the behavior of mathematical arguments and that of arguments that employ degree-vague concepts differ when concatenation of arguments is involved, as is demonstrated by the Sorites.

My solution to the Sorites has some implications for *formal* validity. Suppose we have a sequence of particulars a_1, a_2, … a_N. Suppose further that for some predicate 'F' and for any

$1 \leq n \leq N$-1, if Fa_n then Fa_{n+1}, and that Fa_1. We used to think that it follows that for any $1 \leq n \leq N$, Fa_n. That is, we used to think that every argument of the above form is valid in virtue of its form. However, the paradox of the heap, if my solution is correct, shows that this is not the case, since not all arguments of this form are valid. Let us substitute a vague predicate for 'F', one that applies to a_1 but not to a_N, its vague boundary being somewhere between the two. Then, for sufficiently large n's, namely those beyond the vague boundary, we get an invalid argument: the above two suppositions will be true while the conclusion false. Of course, arguments of the above form are always valid in mathematics, since no degree-vague concept is used in mathematical proofs. (But some mathematical concepts do display other varieties of vagueness—we mentioned the concept of number above.) Accordingly, if we limit ourselves to arguments that do not use degree-vague concepts, we can still maintain that all possible arguments of the Sorites form are valid, and therefore formally valid. But we cannot maintain unconditionally that arguments of this form are valid.

For rejecting the principle that if Fa_1, and if for any n, if Fa_n then Fa_{n+1}, then for any n, Fa_n, the logic of vague concepts can be considered a case of non-inductive logic—'induction' here used as in mathematics. I hope to show, on some other occasion, that non-inductive logic has other independent and significant applications. These additional applications will demonstrate that the principles used in this work to dissolve the Sorites are not *ad hoc*. (And of course, people who live in epistemic and other *ad hoc* glass houses shouldn't throw that stone at others.)

How should those who know that the concept of heap is vague and who accept my solution of the paradox respond, if the propositions of the Sorites argument are put to them one by one, in the attempt to force them to accept the argument's false conclusion? Before discussing that, we should note that the way one would *actually* respond to such questions does not determine the meaning of 'heap' (a comment on recent 'experimental philosophy'). This question & answer game constitutes exceptional circumstances compared to those in which the concept of 'heap' has previously been used. The way one would use it in these new circumstances may reflect a *decision* on behalf of the respondent about the way the concept should be *extended* to these circumstances. The respondent's decision *may* reveal something about the meaning of 'heap' as originally used, but it is not part of that use.

Let us return, then, to the question, how *should* one respond in the question & answer game? Given the solution suggested in this work, respondents have no reason not to accept any single modus ponens step, but they have a good reason not to accept many of them consecutively—the latter, but not the former, may lead from truth to falsehood. But that means that they have a good reason not to participate in the game, apart from, perhaps, a few first steps. There is no use in participating and looking for some sufficient reason to stop at any particular step: there is none. The game as a whole is objectionable, and should be avoided.[4]

But what if one is *forced* to participate in the game? If the propositions of the Sorites argument are put to the respondents one by one, and they have no choice but to reply—if the questioner holds a gun to their head, so to say? Well, the answers rational respondents would give in such circumstances need not reveal what they really think of the matter in hand; they should rather do their best not to get shot. And if one *has* a choice, one should avoid the game.[5]

Solutions like mine, or suggestions along similar lines, have only very rarely been mentioned in the literature, and I am not familiar with any author who developed such a solution in any detail. This may be because of my solution's being, despite its simplicity, quite radical. Some writers, however, did mention similar suggestions and raised a few objections. I shall now try to meet these objections, and also clarify my solution by contrasting it with the few related earlier suggestions with which I am familiar.

The earliest mention I found of an approach similar to mine is in Michael Dummett's

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[4] The response I recommended resembles the one suggested by Chrysippus to the version of the Sorites discussed in antiquity. According to Chrysippus, when asked such questions about successive steps in the sequence, the wise man will stop and keep silent (*hēsychazein*) before he gets to the problematic cases (Cicero, *Academica* II (Lucullus), xxix, § 93; Sextus Empiricus, *Adversus Mathematicos* 7.416). But Chrysippus's justification of his *hēsychazein* may have been insufficient, as is suggested by Carneades's criticism (Cicero, ibid., 93-4).

[5] The following anecdote about Wittgenstein, which I have not seen in print, seems appropriate. I reproduce it as recounted by Peter Hacker, who had heard it from Charles Stevenson, one of Wittgenstein's pupils in the early 1930s: Some of the young Fellows of Trinity, tired of Wittgenstein's constant refrain of 'it depends what you mean', accosted him one day in the Great Court of the College. 'Wittgenstein', they said, 'we have a question for you. But there is one condition: none of your "It depends what you mean". We want a straight Yes or No answer.' 'All right' Wittgenstein replied, 'what is the question?' 'The question is this', they said, 'Legend has it that when the Turk besieged Vienna, and it looked as if the city was going to fall, the walls of the city wept. Now, Wittgenstein, our question is this: Is it true that the walls of the city wept? And mind you, none of your "It depends what you mean"!' 'If you put it that way' Wittgenstein responded, 'then, *yes*, it is true.' 'But how can you say that?', they replied in astonishment. 'If I had said that it isn't', Wittgenstein answered, 'then you would have taken me seriously.'

“Wang’s Paradox” (p. 252). Dummett writes:

> The only alternative left to us […] therefore appears to be to deny that, in the presence of vague predicates, an argument each step of which is valid is necessarily itself valid. This measure seems, however, in turn, to undermine the whole notion of proof (= chain of valid arguments), and, indeed, to violate the concept of valid argument itself, and hence to be no more open to us than any of the other possibilities we have so far canvassed.

Dummett has two worries about this suggestion. I have already discussed and dismissed his second worry, namely, that this suggestion seems ‘to violate the concept of valid argument itself’. His other worry is that this suggestion seems ‘to undermine the whole notion of proof (= chain of valid arguments)’. Keefe and Smith, who, like Dummett, devote a single paragraph to this approach (p. 11), also refer to Dummett’s criticism in order to reject it:

> As Dummett notes, this is to deny the transitivity of validity, which again looks a drastic move given that chaining inferences is normally taken to be essential to the very enterprise of logical proof.

(Cf. also Keefe [2000], p. 20.)

I admit that a result of my solution is that not any chain of valid arguments is a proof of the chain’s final conclusion—the Sorites argument, I have claimed, is not such a chain. But this result should not be seen as undermining the notion of proof. The chains of valid arguments that do not constitute proofs are not an unmanageable set, but can be characterized precisely enough. As I have shown, they are concatenations of valid arguments that pass the vague boundaries of a degree-vague concept they employ. A few variations on the modus ponens version of the Sorites argument brought above are possible, and accordingly some additional kinds of concatenation of valid arguments should also be recognized as yielding invalid arguments (e.g., ones using disjunctive syllogism). But the only chains of valid arguments that according to my solution are not proofs are Sorites chains. We should distrust only arguments that contain a series of sub-arguments applying a degree-vague concept along its tolerance dimension in the way described above, and a few additional arguments that derive their apparent validity from the former (see above). But the validity of all other chains of arguments is unaffected by these considerations (and *in practice* no one actually relies on Sorites chains, so no reasoning we *actually* employ is compromised by these considerations). Most importantly, the validity of mathematical proofs, as has already been explained above, is not endangered thereby, since these do not employ degree-vague concepts. Dummett's apprehensions are therefore unjustified.

Dummett next turns to a discussion of the strict-finitist's approach to vagueness. Although the solution I propose here is not a strict-finitist one, Dummett's objection to strict finitism (p. 253) can be modified so that it apply to my solution as well. Let us therefore examine his objection, appropriately modified.[6]

According to the approach developed in this paper, some concatenations of valid modus ponens arguments of the kind we considered might yield a long invalid Sorites argument. But obviously, to get an invalid Sorites argument the concatenation has to consist of *several* applications of modus ponens. Let us therefore call a number *n non-sorital* if there is no invalid Sorites argument of the kind considered in this paper containing *n* applications of modus ponens. 1, for instance, is non-sorital, for a single application of modus ponens yields a valid argument. 999, by contrast, would be *sorital*, for, as we saw in our example, there are invalid Sorites arguments with 999 applications of modus ponens. If a number is non-sorital then so are all numbers smaller than it, and also, presumably, its successor. Let us choose some sorital number M, which is of course greater than all non-sorital numbers. There is some number k such that M-k is non-sorital (for instance, k=M-1). Now let us call a number *n small* if n+k is non-sorital. Since the successor of a non-sorital number is itself non-sorital, the successor of a small number is itself small. We can now construct the following proof:

1. 0 is small. If 0 is small, then so is 1. Therefore 1 is small.
2. If 1 is small, then so is 2. Therefore 2 is small.

…

($M-k$). If $M-k-1$ is small, then so is $M-k$. Therefore $M-k$ is small.

But if M-k is small, then, by definition, (M-k)+k, namely M, is non-sorital; yet we chose M so that it be sorital. A contradiction, so the distinction between sorital and non-sorital numbers is

[6] Ran Lanzet drew my attention to the applicability of Dummett's objection to the solution suggested in this paper.

apparently untenable. But the approach we developed in this paper is committed to this distinction; so apparently it should be rejected.

But Dummett's argument is unsound. The flaw in it is with its assumption that for some sorital M, some k is such that both M-k is non-sorital and *k itself is non-sorital*: this assumption was built into our definition of a small number. Yet in our example of a k such that M-k is non-sorital, k was equal to M-1; and in that case k is obviously not non-sorital. And I cannot see any reason why one should concede to Dummett the existence of an M and a k of the kind he needs. In his original example, Dummett vacillated between using for that purpose the symbol k for an unspecified number, and the specific number 100 (which qualified as small for his original, different example). That inconsistency in symbolism helped there generate the illusion that finding such a number 'is surely possible'; but once it is removed the persuasive appearance of the argument is eliminated as well. We cannot therefore find in Dummett's work a good argument against our approach.

Paul Ziff (1974) maintained that the modus ponens argument of the Sorites is 'a perfectly good inference to make if you don't make it too often' (p. 530). Sorensen (1988, pp. 46-7) rejects Ziff's position because he thinks it is 'trivial. In order to know whether I have gone too far, I must already know the truth value of the conclusion.' This criticism can be applied to my solution of the paradox as well. However, not only is *Ziff's claim* compatible not only with my solution but with all other suggested solutions of the Sorites, *Sorensen's criticism* is applicable to all these solutions too. Consider, for instance, the epistemic solution, adopted by Sorensen himself. According to it, if we start with a thousand grain heap, for some specific n, 1000-(n-1) grains make a heap while 1000-n grains do not; we just do not know, and perhaps cannot know, which n that is. Accordingly, although our second assumption above, that if a single grain is removed from a heap the remaining collection of grains is still a heap, is false according to Sorensen's epistemic approach, all but one of its instantiations (Nb), $1 \leq N \leq 999$, are true, the false one being (nb). We can therefore, according to the epistemic solution, validly follow the long Sorites argument until we get to conclusion (na), but no further. So, in order to know how many steps in the long Sorites argument are permissible, we should know for which Ns, 1000-N grains make a heap; and 'in order to know whether I have gone too far, I must already know the truth value of the conclusion.'

But apart from this *ad hominem* reply, it would be misleading to consider my solution trivial. It does characterize the invalid Sorites arguments, and thus indicates on which occasions inductive arguments and concatenations of arguments can still be validly used. And besides, it is indeed trivially true that we should know whether the vague boundary has been crossed in order to know whether the argument is invalid.

Rohit Parikh, in pages 259-60 of his "The Problem of Vague Predicates", suggested a solution apparently similar to mine. Parikh notes that if 'you don't argue endlessly' you can avoid contradiction, and adds that indeed 'it is rare in ordinary life for people to make arguments which take a thousand or more steps. Perhaps this is the explanation', he continues, 'of why we use vague predicates in daily life without any serious problem and still avoid difficulties which a logician might run into.' Parikh thus concludes that in logical systems for vague predicates we should 'keep our arguments reasonably short'. This seems similar to my conclusion. However, in order to justify limitations on permissible length of arguments, Parikh finds it plausible to adopt semantics 'in which statements received truth values between 0 and 1.' In that case, 'an application of a rule of inference would allow some small decrease in the truth value so that a very long chain of inferences would no longer be a convincing argument.' By adopting degree-of-truth semantics Parikh makes his approach quite dissimilar to the one developed here.

The only other author I know who discussed an approach similar to mine is Stephen Weiss (1976). Like me, Weiss maintained that some inductive arguments are invalid, specifically those involving vague concepts. (Weiss's discussion is more elaborate, but in ways which do not concern us here.) However, the long Sorites argument formulated above is *not* an inductive argument, and thus Weiss's solution does not apply to it. And inductive arguments, I have claimed, derive their seeming validity from the possibility of substituting concatenated arguments for them. Weiss's solution is therefore inapplicable to the basic cases of Sorites arguments. His solution is therefore insufficient.

Accordingly, I am not acquainted either with an effective criticism of my suggestion in the literature, or with any developed anticipation of it.

When one realizes how the paradox of the heap is resolved, one sees that the solution has been staring us in the face all along. Being captivated by a one-sided, mathematical picture of what arguments are like prevented us from realizing it, and in fact generated the paradox. The solution lies in taking the logic of vague concepts at face value, and leaving everything as it is.[7]

[7] Many friends and colleagues have made many comments on earlier versions of this paper, which has also been

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presented at a few conferences. Their list would pass the vague boundary of the length of acknowledgments appropriate on such occasions, so I hope its omission would be excused.

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