A CRITIQUE OF FREGE ON COMMON NOUNS

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Abstract
Frege analyzed the grammatical subject-term '*S*' in quantified subject-predicate sentences, '*q S* are *P*', as being logically *predicative*. This is in contrast to Aristotelian Logic, according to which it is a *logical subject*-term, like the proper name '*a*' in '*a* is *P*' – albeit a plural one, designating *many* particulars. I show that Frege's arguments for his analysis are unsound, and explain how he was misled to his position by the mathematical concept of function. If common nouns in this grammatical subject position are indeed logical subject-terms, this should require a thorough reevaluation of the adequacy of Frege's predicate calculus as a tool for the analysis of the logic and semantics of natural language.

1. Grammatical subject-terms: Frege's analysis

According to Aristotelian Logic, while some propositions have a single individual as subject, others have as subjects *many* individuals. As an example of propositions of the former kind Aristotle mentioned the sentence 'Socrates is white'; his examples of the latter include 'Every man is white', 'No man is white', 'Some men are white' and 'Not every man is white' (*On Interpretation*, chap. 7). And the subject of 'Every man is white' is a plurality not by virtue of the quantifier 'every', he emphasized, but by virtue of the plural subject-term or universal, 'man' in his examples (ibid., 17b12). Although Aristotle says the subject of the latter is a universal (Platonic residues?), *we* can interpret his *logic* as committed not to reference to universals as some kind of abstract entities, but to reference to many individuals. In sentences of the form '*q S* are *P*' – where '*q*' is any quantifier – the common noun '*S*' designates *many individuals*; it is a *plural referring expression.*

This view of quantified subject-predicate sentences was rejected by Frege. Grammatical plural subject-terms, unlike definite singular ones (e.g., proper names), are not *logical* subject-terms; logically, claimed Frege, they are predicative. This analysis exists already in his *Begriffsschrift*, of 1879. In section 12 he translates the

four Aristotelian quantified subject-predicate sentences, 'All/some S are/aren't P', into his calculus, translations that became canonical with its spread. In these translations, *both* subject-term and predicate are translated by a one-place predicate- or function-symbol. E.g., 'All S are P' is translated by '$\forall x(Sx \rightarrow Px)$' and 'Some S are P' by '$\exists x(Sx \,\&\, Px)$'.

This analysis of Frege's, which was almost unanimously accepted, is, I think, one of his major departures from Aristotelian Logic. (All the same, it is commonly insufficiently appreciated, if not totally unnoticed: it is unmentioned, for instance, in Dummett's entry on Frege in the new Blackwell's *Companion to Analytic Philosophy.*) As such, it surely needs justification. Moreover, it is not self-evident. On the contrary: when one says, for instance, 'Some students were late', 'Many citizens didn't vote' or 'Three children fell asleep', it seems, at least *prima facie*, that one uses the grammatical subject-terms 'students', 'citizens' and 'children' not predicatively, but to *refer* to a plurality of individuals. Again, a view to the contrary needs justification. How did Frege, then, justify his analysis?

2. Frege's arguments: exposition and criticism

Although this analysis appears, as was mentioned above, already in the *Begriffsschrift*, Frege argues in its support only in later writings. The earliest place I found is in his *Die Grundlagen der Arithmetik*, of 1884. Section 47 of the *Grundlagen* contains the following argument, intended to show that in the sentence 'All whales are mammals' we do not speak about whales – i.e., 'whales' does not refer to whales – but about the *concept* of a whale:

> Indeed, the sentence [*Satz*] 'All whales are mammals' seems at first sight to be about animals and not about concepts; but if one asks, which animals then are talked about, not a single one can be produced [*aufweisen*].

Of course, when we talk about whales, there is usually no whale around to which we could point. But all the same, the answer to the question, which animals are talked about, is straightforward: whales. So what does Frege mean by his comment that 'not a single one can be produced'? He continues as follows:

> Suppose a whale were present, then nevertheless our sentence would not claim anything about it. One could not infer from

our sentence that the animal present is a mammal without adding the sentence that it is a whale – something our sentence does not contain at all [*wovon unser Satz nichts enthält*].

The intended conclusion, that the sentence is not about whales, does not follow from this argument. If one said 'Peter is ill' and Peter were present, one could not infer that the man present were ill without adding that he is Peter – 'something our sentence ["Peter is ill"] does not contain at all'. Yet 'Peter is ill' is about Peter, even according to Frege. Analogously, 'All whales are mammals' can still be about whales, even if one does not know of a certain animal that it is a whale.

It seems that Frege confuses epistemology and semantics in this case. A sentence can be about certain particulars without the speaker's being able to say of a given particular in specific circumstances that it is among those the sentence is about – and this is the case with proper names as well as with common nouns.

Frege concludes this part of his discussion thus:

Generally it is impossible to speak of an object without designating [*bezeichnen*] or naming [*benennen*] it in some way. But the word 'whale' names no particular.

This, I think, begs the question. On the one hand, we can say that 'whale' does designate all particular whales. On the other, we can say that although it does not specifically name any particular whale, it is used here to speak of all whales. Frege fails to supply us here with a good justification of his position.

Frege considered conclusive this argument against the view that the common noun in quantified subject-predicate sentences designates a plurality of particulars. A decade later he refers back to section 47 of the *Grundlagen*, in his review of Husserl's *Philosophie der Arithmetik* (p. 327; p. 83 in the English translation), and says that he has there adequately demonstrated (*hinlänglich dargelegt*) the point. However, he adds in this review another argument against that view, an argument which he repeated a year later in his 'Kritische Beleuchtung einiger Punkte in E. Schröders *Vorlesungen über die Algebra der Logik*' (p. 454; p. 105 in the English translation). In the former review he writes:

It is surely clear that by means of the sentence 'all men are mortal' one does not want to assert anything about some Chief Akpanya, of whom perhaps he has never heard.

And in the latter:

> If I utter a sentence with the grammatical subject 'all men', I do not wish to assert by it anything about some Central African chief wholly unknown to me. It is thus utterly false that I am in any way designating this chief with the word 'man', that this chief belongs in any way whatsoever to the reference (*Bedeutung*) of the word 'man'.

But isn't Frege himself, who time and again warned us against conflating psychology and semantics, making this very mistake here? Of course, when I say 'All men are mortal', many people – Chief Akpanya included – are not before my mind. But I certainly assert something about them: that they are mortal. If I were asked, whether what I said applies to Chief Akpanya as well, I would answer affirmatively, although I had never heard of him until then. Whether or not I knew him, I asserted something about that man, Chief Akpanya, when I uttered the sentence 'All men are mortal'. So it seems that I *am* designating the chief or referring to him when I use 'men' as subject-term in that sentence. (Notice that according to Aristotelian Logic '*men*' is the logical subject-term, and not 'all men'; the quantifier is not part of the logical subject-term.)

In 'Über Begriff und Gegenstand', of 1892, Frege brought two additional arguments for taking common nouns to be predicative and not logical subject-terms, even when in the grammatical subject position (pp. 197–8). Firstly, he writes that even in a sentence like 'All mammals have red blood' the predicative nature of 'mammals' cannot be mistaken, since that sentence can be paraphrased as 'Whatever is a mammal has red blood' or 'If anything is a mammal, then it has red blood'.

But first, assuming that paraphrases always employ the same concepts and in the same way, one can use paraphrases both ways: why not say that the first sentence, where 'mammal' apparently functions as a logical subject-term, shows that in the paraphrases 'mammal' is *not* used as a logical predicate, appearances notwithstanding? An additional reason is needed in order to justify taking the paraphrases as revealing the nature of the concepts in the paraphrased sentence, and not the other way around. Secondly, we should in fact *reject* that assumption: two sentences can say the same thing although using concepts in different ways. For instance, sentences of the form '$p \mathbin{\&} q$' and '$\neg(\neg p \mathbin{V} \neg q)$' are

logically equivalent, yet in the former we are using neither a disjunction nor negation. And lastly, it is doubtful whether 'All mammals have red blood' and 'If anything is a mammal, then it has red blood' *are* equivalent: unless we are already committed to Frege's predicative analysis of grammatical subject-terms, it seems plausible to maintain that the first sentence, but not the second, presupposes reference to mammals.

Frege's second argument in 'Über Begriff und Gegenstand' for the same conclusion is as follows. If in 'All mammals are land-dwellers' the phrase 'all mammals' expressed the logical subject of the predicate 'land-dwellers', then its negation should be 'All mammals are not land-dwellers', which it is not. Accordingly, 'all mammals' does not express the subject of that sentence. Frege repeated this argument in a note on page 441 of his 'Kritische Beleuchtung' (p. 93 in the English translation). He there used as an example the sentence 'All bodies are heavy' and argued that from the fact that the negation of this sentence is 'Not all bodies are heavy' it follows that neither 'all bodies' nor the concept *body* are the subject of that sentence.

But why should the negation of 'All mammals are land-dwellers' be of the form Frege specifies in case 'mammals' designates its subject? The negation of 'Peter and Mary are painters' is not 'Peter *and* Mary are not painters' but 'Peter *or* Mary is not a painter' or 'Peter and Mary are not both painters'. Yet 'Peter and Mary' designates the subject of that sentence: 'Peter and Mary' is used to specify two people of whom something is said in the sentence. All that these examples show is that the syntax of a negation of a sentence with a plural subject is not that of the negation of a sentence with a singular subject.

Here one might argue – with Frege – that the sentence 'Peter and Mary are painters' is actually not a subject-predicate sentence with a plural conjunctive subject-term, but a kind of a contraction of a conjunction of two sentences, 'Peter is a Painter and Mary is a Painter'. In a letter to Russell (dated 28 July 1902), and later in his posthumously published 'Logic in Mathematics', written in 1914, Frege claimed that in such sentences 'we are not really connecting the proper names by "and"', but telescoping two connected sentences into one ('Logic in Mathematics', p. 227); his examples being 'Schiller and Goethe are poets' (ibid.) and 'Socrates and Plato are philosophers' (Letter to Russell; cf. also his *Grundlagen*, note to section 70). This contraction is being made for the sake convenience (Letter to Russell).

I cannot see how this view of the sentence with the conjunctive subject-term can make any difference. According to it, a sentence of the form (1) '*a* and *b* are *P*' is equivalent to the sentence (2) '*a* is *P* and *b* is *P*'. But (2), according to Frege as well, is a conjunction of two subject-predicate sentences, the first having *a* as its subject, the second *b*. So although (2) is not a subject-predicate sentence, and therefore it has no logical subject-*term*, it has two particulars as *subjects*, *a* and *b* – subjects in the sense that they are the two particulars, specified by some phrases in the sentence, about which something is being said by the sentence. Consequently, since (1) and (2) were supposed to be equivalent, (1) has two particulars as subjects as well, designated by its grammatical subject-term, '*a* and *b*'. And the negation of (1) is not '*a* and *b* are not *P*'. So the negation of a sentence with a plural subject is not of the same form as the negation of a sentence with a singular one. Our counter-example to Frege's argument still stands.

But even if our counter-example were rejected, Frege's argument would still be invalid: there is no necessity that the form of a negation of a sentence should be uniform for sentences with singular and plural subjects.

I did not find any other argument in Frege's writings against the view that common nouns in quantified subject-predicate sentences are used as logical subject-terms. And all the arguments mentioned above are, I argued, unsound. I therefore think that Frege failed to show that these common nouns are not used as logical subject-terms.

3. The mathematical function: Frege misled

How was Frege led to his unjustified analysis of these common nouns? As was mentioned above, this analysis appears already in his *Begriffsschrift*, although he argues for it only in later writings. It seems these arguments were intended to justify a position arrived at due to other factors.

As the subtitle of Frege's *Begriffsschrift* declares, his formula language was 'modeled upon the formula language of Arithmetic'. Under the influence of mathematics, Frege interpreted the subject-predicate relation as an argument-function relation. Now in mathematics, only terms designating a single entity occupy the argument place in the functional notation. For instance, in

'$f(x) = x^2$' we can substitute only a term designating a single number for the variable 'x'; e.g., '3' as in '$f(3) = 3^2$'. And in '$f(x, y) = x + y$' we substitute two terms, each designating a single number; e.g., '1' and '2', as in '$f(1, 2) = 1 + 2$'. Consequently, when Frege interprets natural language sentences as incorporating the function-argument relation (ibid., § 9), he is led – without any justification – to assume only singular names as arguments.

For instance, one of his examples there is the sentence 'The circumstance of carbon dioxide's being heavier than hydrogen'. In that case he considers as arguments 'carbon dioxide' and 'hydrogen', each term denoting a *single* gas. Another example he mentions there is 'Cato killed Cato': 'Cato', again a word designating a *single* person, is thought of as argument, in its first occurrence, in its second one, or in both.

But Frege could have equally well mentioned there as examples sentences with *plural* subjects. 'Brutus and Cassius killed Cesar', 'The senators killed Cesar', and so on. We may think of the first subject-expression as variable, and substitute in its place either an expression designating a single individual (e.g., 'Brutus') or an expression designating *several* individuals (e.g., 'Brutus and Cassius', 'The senators').

Frege's functional analysis of sentences does not in itself justify confining the argument place to expressions designating singular items. The fact that only such expressions function as arguments in mathematics is an insufficient reason for limiting natural language's arguments in the same way.

I therefore think that while developing his *Begriffsschrift*, Frege was led unawares, under the influence of the mathematical concept of function and argument, to confining the argument or logical subject-term role to singular referring expressions. Only in later writings did he try, unsuccessfully, to justify this position, to which his developed *Begriffsschrift* already committed him.

It is still possible to argue, however, that Frege's analysis was justified *post factum*: his concept-script supplied a very powerful tool for the analysis of the semantics and logic of natural language, much more so than did Aristotelian Logic; and its success certainly constitutes a good reason for presuming the correctness of the analyses on which it is grounded.

The reply to this argument should consist, first, in an attempt to show that on the whole, relevant linguistic phenomena are explainable more successfully by the analysis of common nouns in subject position as plural referring expressions; and, secondly,

in an attempt to develop a deductive system for natural language, grounded on that analysis and similar in its power to Frege's *Begriffsschrift*. This is obviously beyond the scope of this paper. I did attempt to do this in a different publication (Ben-Yami 2004); but the more modest purpose of this paper was to show that Frege failed to justify his claim that common nouns in the subject position in quantified subject-predicate sentences are not logical subject-terms, designating a plurality of particulars.

Assuming, however, that these attempts are successful, the consequences for the place of Fregean logic in the study of the semantics and logic of natural language would be far-reaching. Fregean logic would not only be shown to be grounded on an unjustified semantic analysis, but also to be inessential for this study, and perhaps even inferior to other systems in its power to analyze and explain various linguistic phenomena. The present paper did not, of course, establish these results; but it did make, I hope, a step in that direction.

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